OSCAR: Integrating GAP and Julia

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1 Introduction to OSCAR





All of this is joint work with

- William Hart (TU Kaiserslautern)
- Thomas Breuer (RWTH Aachen)
- Reimer Behrends (TU Kaiserslautern)
- Max Horn (JLU Gießen)
- Markus Pfeiffer (University of St Andrews)

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For a complete list of people involved in the various parts of OSCAR, see https://oscar.computeralgebra.de/credits.



② GAP-Julia Integration



Overview



Multigraded equivariant Cox rings of toric varieties over number fields



Graphs of groups in division algebras

Matrix groups over polynomial rings with coefficients in number fields

Gröbner fans over fields with discrete valuations

polymake: convex polytopes, polyhedral and stacky fans, simplicial complexes and related objects from combinatorics and geometry.



ANTIC: number theoretic software featuring computations in and with number fields and generic finitely presented rings.

eilul

Gutsche OSCAR: Integrating GAP and Julia

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- make all functionality from each system available in every other system;
- make all systems share a common mid-level programming layer.

We use Julia as a powerful mid-level programming layer.

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- possibility to extend systems with Julia code, making use of Julia's powerful JIT-compiler, type system, and extensive library.

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- GAP: Second part of talk

Example for using Nemo number fields as coefficient rings in Singular

Julia_rings_with_Singular.ipynb

All information about the OSCAR project can be found on

https://oscar.computeralgebra.de

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On the page you can find

- news,
- blog posts,
- examples,
- and installation instructions.







JuliaInterface and GAP.jl

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https://github.com/oscar-system

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Possible conversions:

- Integers
- Floats
- Strings
- Booleans
- Nested lists of the above to Arrays or Tuples

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gap> ImportJuliaModuleIntoGAP( "Base" );
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gap> Julia.Base.sqrt( 4 );
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- Julia functions can be used like GAP functions
- If necessary and possible, input data is converted to Julia
- Calling only possible for convertible types and Julia objects

Using JuliaInterface, it is possible to write Julia functions and use them as GAP kernel functions (from orbits.jl):

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```
function orbit( element, generators, action )
 work set = [ element ]
 return set = [ element ]
 generator_length = gap_LengthPlist(generators)
 while length(work_set) != 0
    current element = pop!(work set)
   for current_generator_number = 1:generator_length
      current_generator = gap_ListElement(generators,
                                          current generator number)
      current result = gap CallFunc2Args(action.current element.
                                         current_generator)
      is in set = false
      for i in return set
       if i == current_result
         is in set = true
          break
       end
      end
      if ! is in set
        push!( work_set, current_result )
        push!( return_set, current_result )
      end
    end
  end
 return return set
end
```

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Compiled Julia functions come close to the performance of kernel functions:

gap> S := GeneratorsOfGroup(SymmetricGroup(10000));;

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gap> S := GeneratorsOfGroup( SymmetricGroup( 10000 ) );;
gap> orbit_gap( 1, S, OnPoints );; time;
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gap> orbit_jl( 1, S, OnPoints );; time;
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gap> orbit_c( 1, S, OnPoints );; time;
46
```

julia> S3 = GAP.SymmetricGroup(LibGAP.to_gap(3))
GAP: SymmetricGroup([1 .. 3])

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julia> size_gap = GAP.Size( S3 )
GAP: 6
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GAP: SymmetricGroup( [ 1 .. 3 ] )
```

```
julia> size_gap = GAP.Size( S3 )
GAP: 6
julia> LibGAP.from_gap( size_gap, Int64 )
6
```

From the GAP side

How does GAP benefit from Julia/OSCAR (except mathematical algorithms)?



Speedup

• Now: Find time critical parts of algorithms, rewrite them in C.

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Language features

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- Built-in traits: Known properties of objects decide which variant of an algorithm to use
- Immediate propagation: Second execution layer is used to spread properties between objects
- Categorical programming language as defined in the CAP project

Example for using Singular in GAP via Julia

Using Singular from GAP.ipynb

Introduction to OSCAR

② GAP-Julia Integration



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- it adds a layer of indirections and causes inefficiencies and
- unreachable cycles that involve both GAP and Julia objects cannot be reclaimed, so it leads to memory leaks.

Using the same GC for GAP and Julia

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- This way, all GAP objects are first-class citizens in Julia, and Julia objects are first class citizens in GAP
- Thus using Julia objects from GAP and GAP objects from Julia works without any GC overhead (essentially no penalty at all)





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